Disentangling and quantifying market participant volatility contributions

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Joint work with J.F. Muzy and M.Rambaldi

The 1-Dimensional Poisson process

- N_t : jump process (jumps are all of size 1)
- λ_t : the intensity
- μ : 1-dimensional exogenous intensity

$$\lambda_{\mathbf{t}} = \mu$$

$\Longrightarrow$ **The inter-arrival times are independant**

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A Hawkes process

$\Longrightarrow$ Introducing (positive) correlation in the arrival flow
$\Longrightarrow$ "Auto-regressive" relation

$$\lambda_{\mathbf{t}} = \mu + \phi \star \mathbf{dN_t},$$

where by definition

$$\phi \star dN_t = \int_{-\infty}^{+\infty} \phi(t-s) dN(s)$$

and $\phi(t)$: kernel function, positive and causal (supported by R^+).

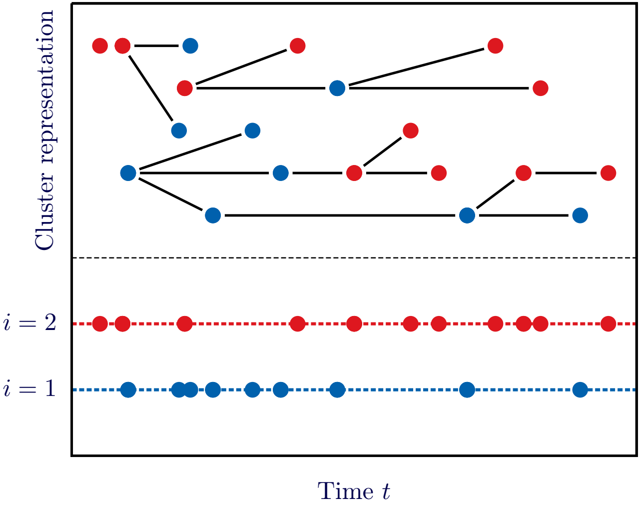
- N_t : a D-dimensional jump process (jumps are all of size 1)
- λ_t : D-dimensional stochastic intensity
- μ : D-dimensional exogenous intensity
- $\Phi(t)$: $D \times D$ square matrix of kernel functions $\Phi^{ij}(t)$ which are positive and causal (i.e., supported by R^+).
- Moreover $||\Phi^{ij}||_{L^1} < +\infty, \quad 1 \leq i,j \leq D$

"Auto-regressive" relation

$$\lambda_{\mathbf{t}} = \mu + \mathbf{\Phi} \star \mathbf{dN_t},$$

where by definition

$$(\Phi \star dN_t)^{ij} = \sum_{k=1}^{D} \int_{-\infty}^{+\infty} \Phi^{ik}(t-s) dN^k(s)$$


Cluster representation
$i = 2$
$i = 1$
Time t

For each component (we assume stationarity)

$$\Lambda^i = \mathbb{E}\left[\lambda(t)\right] = \mu_i + \sum_{j=1}^{D} \Lambda^j \|\phi^{ij}\|$$

where we define

$$\|\phi^{ij}\| = \int_0^\infty \phi^{ij}(t)dt$$

Hence:

- μ^i is the immigrant intensity of type i events.
- $\|\phi^{ij}\|$ is the average number of type i event triggered by a type j event.
- The shape of $\phi^{ij}(t)$ specifies how the excitation develops in time.

The **Branching ratio matrix** provides a summary of the interactions

$$\boldsymbol{G} = \{G^{ij}\}_{i,j=1,\ldots,D} = \{\|\phi_{ij}(t)\|\}_{i,j=1,\ldots,D}$$

Inference in D-dimensionnal Hawkes processes framework

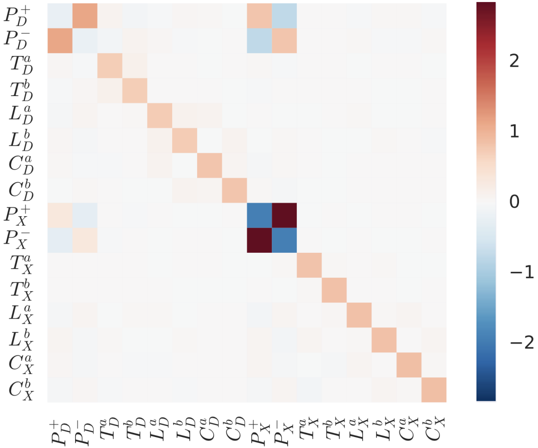
Estimating $D \times D$ real-valued functions

- **Parametric approaches**
  - ϕ^{ij} = linear combination of atomic functions in a dictionnary (e.g., exponential functions with various decay exponents)
  - Many procedures with various assumptions (sparsity, low-rank, ...)

- **Non-parametric approach**
  - Several methods in small dimension but **very difficult task in large dimension !**
  - M.Achab, et al. ICML (2017) JMLR (2017)
    $\rightarrow$ **Direct estimation of $(\boldsymbol{G})^{ij} = \int \phi^{ij}(t)dt$ without estimation of $\phi^{ij}(t)$**

M.Achab, E.B., J.-F.Muzy, M.Rambaldi, Quantit. Finance (2018)

Estimation of **256 kernels** of the **DAX+Eurostoxx Branching ratio matrix**



$\widehat{G}$

The Reaction matrix

$$\boldsymbol{R} = (\boldsymbol{I_d} - \boldsymbol{G})^{-1}$$

- gives the average direct and indirect effect of an event;
- R^{ij} = number of events of type i generated in total by an event of type j
- $\Lambda^i = E(\lambda_t^i) = \sum_j R^{ij}\mu_j$

Let δ_i be the mid-price change determined by an event of type i, then

$$\Delta_\tau P(t) \equiv P(t+\tau) - P(t) = \sum_{i \in M} \delta_i \int_t^{t+\tau} dN_s^i$$

And the volatility at time scale τ :

$$\sigma_\tau^2 = E(\Delta_\tau P^2) = \sum_{i,j \in M} \delta_i \delta_j \int_0^\tau \int_0^\tau E(dN_s^i dN_{s'}^j)$$

Putting together

- $\boldsymbol{R} = (\boldsymbol{I_d} - \boldsymbol{G})^{-1}$
- $\sigma_\tau^2 = \sum_{i,j \in M} \delta_i \delta_j \int_0^\tau \int_0^\tau E(dN_s^i dN_{s'}^j)$

After some calculations one obtains for the diffusive volatility :

$$\frac{\sigma_\tau^2}{\tau} \xrightarrow[\tau \to \infty]{} \sum_m \Lambda^m \xi_m^2 = \sum_{m=1}^{n} \Lambda^m \left(\sum_{i \in M} \delta_i R^{im} \right)^2$$

where

$$\xi_m = \text{average volatility per event of type } m$$

We have a link from microscopic dynamics to the diffusive regime

- $N_{i,\alpha}(t)$ counting process associated with actions α of agent i.
- We will suppose that $i = 1, \ldots, M$ and $\alpha \in \mathcal{A} = \{P^+, P^-, L^a, L^b, C^a, C^b, T^a, T^b\}$ where
  - P^+ (P^-) orders that immediately move upward (downward) the mid-price;
  - T^a (T^b) aggressive orders at the best ask (bid) that do not move the price;
  - L^a (L^b) new limit orders that arrive at the best ask (bid);
  - C^a (C^b) cancel orders at the best ask (bid) that do not move the price;
- $\phi^{i,\alpha;j,\beta}$ = influence of order type β of agent j on order type α of agent i

Total number of interactions: $(M \times 8) \times (M \times 8)$
For $M = 15$ agents that's 14400 kernels to estimate !!

Such huge number of kernels is hard to handle.

↓

Work hypothesis:

- Influence on agent i from agent j actions does not depend on j provided $j \neq i$.
- That is

$$\phi^{i,\alpha;j,\beta}(t) = \begin{cases} \phi^{i,\alpha;\beta}(t) \text{ if } i \neq j \\ \phi^{i,\alpha;i,\beta}(t) \text{ if } i = j \end{cases}$$

Data are labelled data provided by Euronext

- CAC40 index future
- we consider the most liquid expiry for each day
- from March 1st 2016 to February 28th 2017;
- 111 unique members (connections are aggregated);
- focus on equity hours (08:00 - 16:30 London time)

We consider this subset of agents:

- at least 1000 orders at the first level;
- are active "uniformly" between 08:00 and 16:30;
- respect the above for at least 30 days.

Total number of agent considered $M = 16$

Name	Description
End of day (EOD) position	Absolute change in inventory at the end of the trading day, divided by the total volume traded by the agent.
Proprietary	Fraction of the orders that are market as proprietary by the agent.
Order lifetime	Median time between limit order insertion and cancellation/modification.
Inter-event time	Median time between two different orders by the same agent.
Limit-filled	Fraction of the submitted limit orders that are at least partially filled.
Canceled orders	Fraction of limit orders that are eventually canceled.
Aggressive volume	Ratio of the volume traded aggressively over the total traded volume by the agent.
Orders/Trades	Number of orders submitted for each trade.
Order size	Average order size (in contracts).
Time present at L1	Fraction of time the agent was present with a limit at at least one of the best quotes.
Present at both sides	Given the agent was present at the best, fraction of time he was present at both sides simultaneously.
Active connections	Average number of connections used by the agent per day.
Daily volume fraction	Fraction of the total traded volume (total buy + total sell) in which the agent is involved.

Empirical results : Summary characteristics

	240	140	478	127	636	398	503	274	566	59	584	364	597	455	244	669
EOD Position / Volume (%)	0.00	0.01	0.15	3.73	3.83	4.54	9.71	14.9	16.2	22.3	18.3	22.7	24.5	29.1	28.2	32.8
% Proprietary	100.0	100.0	100.0	100.0	100.0	100.0	0.22	68.1	97.8	100.0	1.19	100.0	2.10	98.7	0.00	0.37
Order lifetime (s)	0.51	0.61	0.20	3.57	0.99	0.33	1.33	3.19	42.0	4.14	7.87	5.17	4.32	3.04	6.31	11.1
Inter-event time (s)	0.01	0.00	0.02	0.01	0.00	0.06	0.63	0.07	0.63	0.01	1.64	0.01	1.65	0.12	2.45	2.33
Limit filled (%)	5.09	6.15	8.40	10.5	6.35	10.5	28.3	19.8	47.9	1.58	50.4	5.45	42.4	4.05	23.5	42.0
Limit (%)	51.1	50.0	48.9	44.3	36.3	37.7	49.3	47.5	53.6	40.7	31.0	51.0	54.1	50.0	48.1	53.4
Cancel (%)	48.4	47.2	46.2	40.0	33.7	33.9	36.2	37.9	27.9	40.1	14.5	48.3	31.1	48.0	36.6	30.2
Replace (%)	0.00	0.08	3.43	13.6	29.4	27.4	6.58	8.78	7.54	18.4	40.1	0.04	5.77	1.57	11.1	8.57
Aggressive (%)	0.51	2.69	1.42	2.08	0.60	1.01	7.97	5.76	10.9	0.80	14.4	0.62	9.08	0.43	4.25	7.78
Aggressive volume (%)	14.9	64.0	34.0	34.4	15.0	13.2	49.9	46.9	37.4	56.2	44.4	25.0	34.8	27.0	27.0	28.8
Orders/Trades (%)	3994.2	1085.8	1351.5	1128.0	5573.1	1036.1	238.5	524.7	190.3	5276.6	191.9	2609.4	206.5	3915.7	162.6	497.4
Order size (contracts)	1.02	1.38	2.33	1.65	1.15	4.41	2.45	1.64	1.70	1.88	3.66	2.42	2.70	4.08	3.75	2.38
Time present at L1 (%)	76.8	99.4	51.1	87.6	73.7	26.5	39.3	38.4	22.7	30.4	19.7	36.1	25.1	22.2	27.0	42.6
Present at both sides (%)	39.1	69.1	9.21	36.9	25.9	0.69	4.71	5.07	1.59	1.61	0.99	1.32	1.75	0.69	1.91	5.87
Active connections	19.9	98.2	16.2	32.2	19.6	2.16	18.9	19.8	9.32	5.47	17.7	10.5	4.26	13.9	2.55	3.69
Daily volume fraction (%)	2.22	31.3	4.68	6.30	1.28	3.59	6.05	5.63	2.04	4.76	3.85	2.00	1.88	2.13	2.65	2.73

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At the far left : Flat position, fast, high order to trade ratio, proprietary, high presence at L1.
$\simeq$ **Market maker**

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At the far right :Slower, directional, lower order/trade ratio.
$\simeq$ **Directional agent**

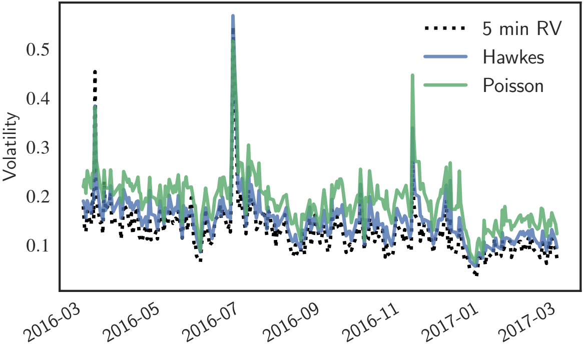
Average direct and indirect contribution to the total volatility of a single event of type $\{i, \alpha\}$:

$$\xi_{i,\alpha}^2 = \left(\sum_j \sum_\beta \delta_{j,\beta} R^{j,\beta;i,\alpha} \right)^2$$

where we assume that $\delta_{j,\beta} = 0$ if $\beta \notin \{P^+, P^-\}$.

And the total diffusive volatility writes

$$\sigma^2 = \sum_{i,\alpha} \Lambda^{i,\alpha} \xi_{i,\alpha}^2$$



$$\text{Hawkes volatility} = \sum_{i,\alpha} \Lambda^{i,\alpha} \xi_{i,\alpha}^2$$
$$\text{Poisson volatility} = \sum_{i,\alpha} \Lambda^{i,\alpha} \delta_{i,\alpha}^2$$

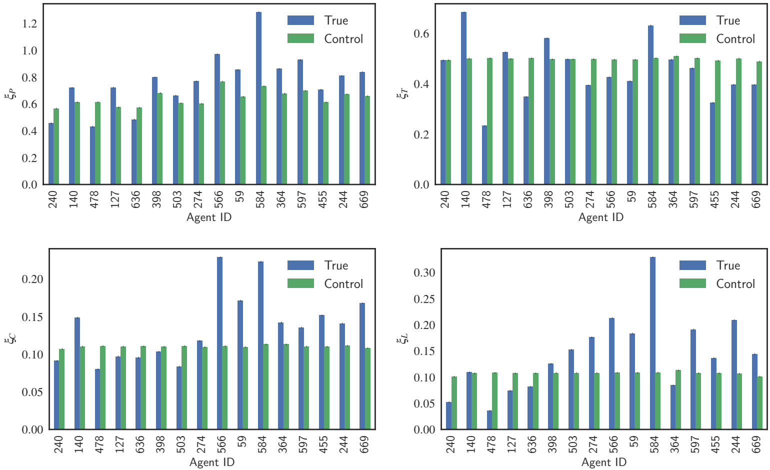
For each agent, we construct a "control group" with 10 "control agents" to compare with.

Every single day :

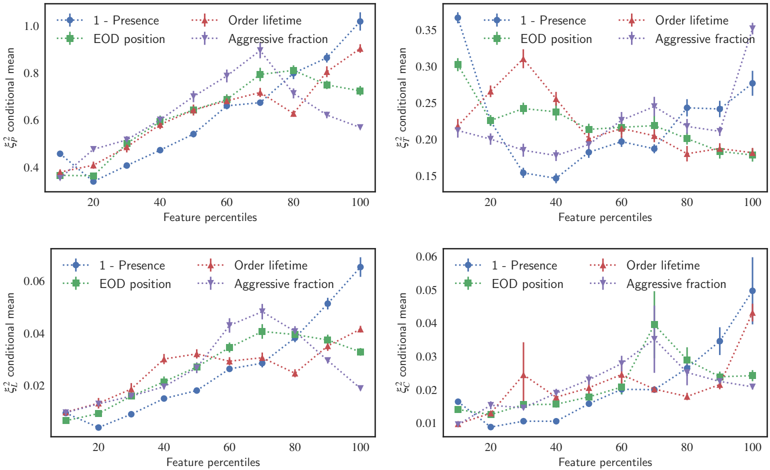
- Each real agent's order is assigned randomly to one of the control agent following the two next rules
- The control agents have the same number of orders
- The control agents have the same order type composition as the real agent

$\Longrightarrow$ **differences between real behavior and control are mainly due to timing.**

Each control value for a given agent is then obtained by averaging on the values of each control agent



ξ strongly depends on the agent order timing



Market maker like agents have smaller impact per passive event

Given

$$\sigma^2 = \sum_{i,\alpha} \Lambda^{i,\alpha} \left(\sum_{j,\beta} \delta_{j,\beta} R^{j,\beta;i,\alpha} \right)^2$$

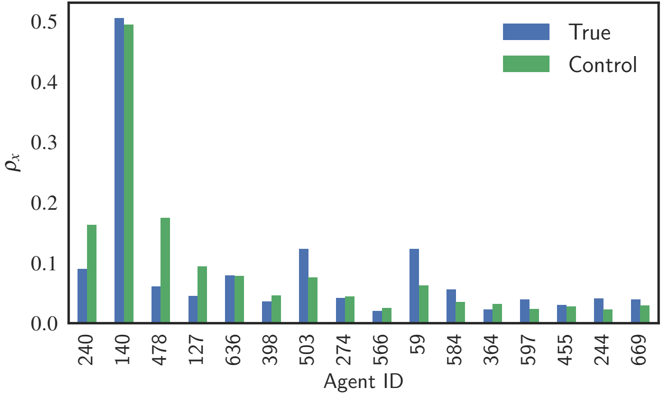
and

$$\Lambda^{i,\alpha} = \sum_{k,\gamma} R^{i,\alpha;k,\gamma} \mu^{k,\gamma},$$

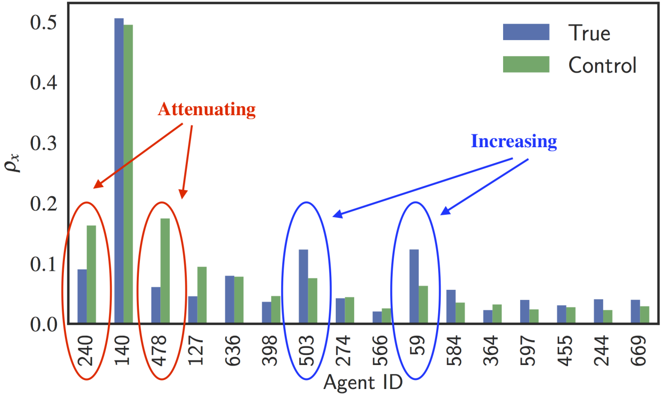
we define ρ_m for agent m as

$$\sigma^2 \rho_m = \sigma^2 - \sum_{i \neq m} \sum_{k \neq m} \sum_{\alpha,\gamma} R^{i,\alpha;k,\gamma} \mu^{k,\gamma} \left(\sum_{j \neq m} \sum_{\beta} \delta_{j,\beta} R^{i,\alpha;j,\beta} \right)^2$$

ρ_m **: Relative difference in volatility we would observe if we removed all the activity directly or indirectly generated by agent** x**.**

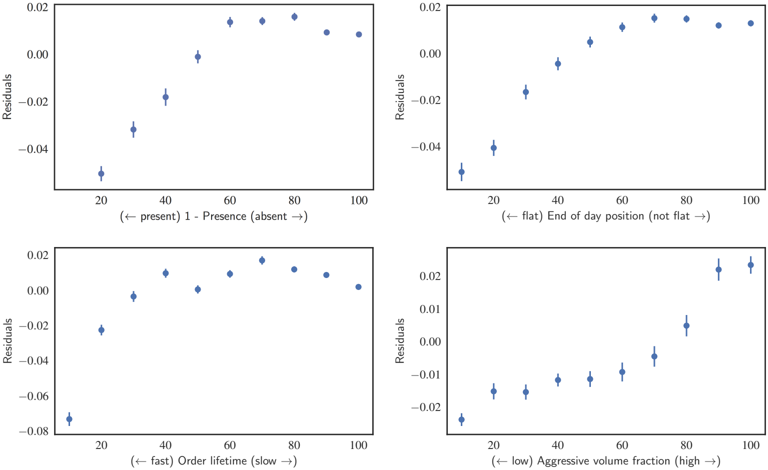


Significant differences with the control for most agents (and $\rho_m > 0$)



Significant differences with the control for most agents (and $\rho_m > 0$)

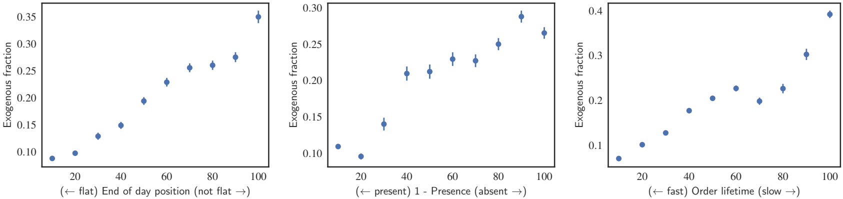
Plotting the residuals : $\rho_m - \rho_m^{\text{control}}$



Market-marker like agent (left side) have volatility-attenuating timing.

Exogenous fraction f_m for agent m

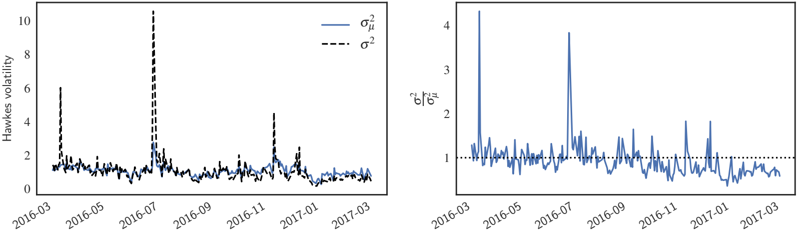
$$f_m = \frac{\sum_\alpha \mu^{m,\alpha}}{\sum_\alpha \Lambda^{m,\alpha}}.$$



Marker maker like (left side) are more “endogenous”.

$$\sigma^2 = \sum_{i,\alpha} \Lambda^{i,\alpha} \left(\sum_{j,\beta} \delta_{j,\beta} R^{j,\beta;i,\alpha} \right)^2 = \sum_{i,\alpha} \mu_t^{i,\alpha} u^{i,\alpha} \approx \sum_{i,\alpha} \mu_t^{i,\alpha} \bar{u}_t^{i,\alpha} = \sigma_\mu^2$$

$u^{i,\alpha}$ = volatility per exogeneous event
$\bar{u}_t^{i,\alpha}$ = mean value over a month.



Good approximation except on extreme days

M.Bompaire, P.Deegan, S.Gaiffas, S.Poulsen, E.B., . . .

- Python 3 et C++11
- Open-source (BSD-3 License)
- `pip install tick` (on MacOS and Linux...)
- `https://x-datainitiative.github.io/tick`
- Statistical learning for time-dependent models
- Point processes (Poisson, Hawkes), Survival analysis, GLMs (parallelized, sparse, etc.)
- A strong simulation and optimization toolbox
- Partnership with Intel (use-case for new processors with 256 cores)
- Many contributors
- New contributors are welcome !

tick 0.1 Home Examples API Browse ▾ Search

tick

tick a machine learning library for Python 3. The focus is on statistical learning for time dependent systems, such as point processes. Tick features also tools for generalized linear models, and a generic optimization toolbox.

The core of the library is an optimization module providing model computational classes, solvers and proximal operators for regularization. It comes also with inference and simulation tools intended for end-users.

Show me »

Examples

Examples of how to simulate models, use the optimization toolbox, or use user-friendly inference tools.

Inference

User-friendly classes for inference of models

Simulation

User-friendly classes for simulation of data

Optimization

The core module of the library: an optimization toolbox consisting of models, solvers and prox (penalization) classes. Almost all of them can be combined